- I can use similarity theorems to prove triangles are similar.
- I can use similar triangles to solve problems, including Thales's shadow method.

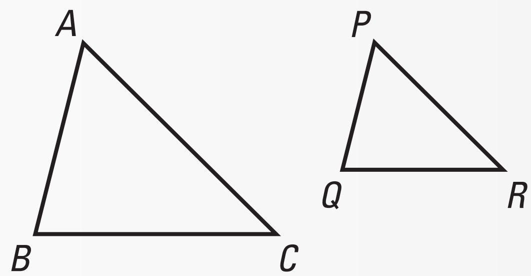
THEOREMS

THEOREM 8.2 ***Side-Side-Side (SSS) Similarity Theorem***

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$,

then $\triangle ABC \sim \triangle PQR$.

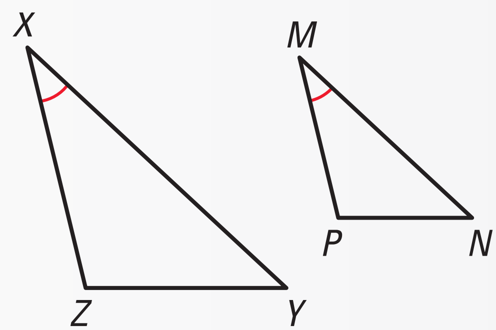


THEOREM 8.3 ***Side-Angle-Side (SAS) Similarity Theorem***

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$,

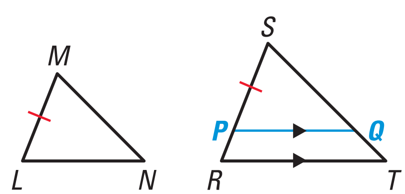
then $\triangle XYZ \sim \triangle MNP$.



EXAMPLE 1 ***Proof of Theorem 8.2***

GIVEN ▶ $\frac{RS}{LM} = \frac{ST}{MN} = \frac{TR}{NL}$

PROVE ▶ $\triangle RST \sim \triangle LMN$



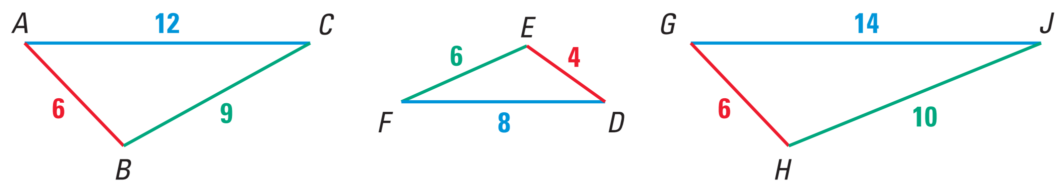
SOLUTION

Paragraph Proof Locate P on $\overline{RS}$ so that $PS = LM$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$, by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$. Because $PS = LM$, you can substitute in the given proportion and find that $SQ = MN$ and $QP = NL$. By the SSS Congruence Theorem, it follows that $\triangle PSQ \cong \triangle LMN$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle LMN$.

EXAMPLE 2 ***Using the SSS Similarity Theorem***

Logical Reasoning

Which of the following three triangles are similar?



STUDENT HELP

Study Tip
Note that when using the SSS Similarity Theorem it is useful to compare the shortest sides, the longest sides, and then the remaining sides.

SOLUTION

To decide which, if any, of the triangles are similar, you need to consider the ratios of the lengths of corresponding sides.

Ratios of Side Lengths of △ABC and △DEF

$\frac{AB}{DE} = \frac{6}{4} = \frac{3}{2}$, $\frac{CA}{FD} = \frac{12}{8} = \frac{3}{2}$, $\frac{BC}{EF} = \frac{9}{6} = \frac{3}{2}$

Shortest sides **Longest sides** **Remaining sides**

▶ Because all of the ratios are equal, $\triangle ABC \sim \triangle DEF$.

Ratios of Side Lengths of △ABC and △GHJ

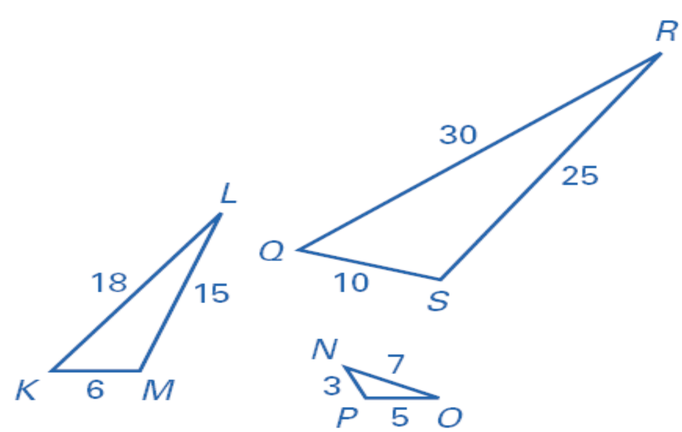
$\frac{AB}{GH} = \frac{6}{6} = 1$, $\frac{CA}{JG} = \frac{12}{14} = \frac{6}{7}$, $\frac{BC}{HJ} = \frac{9}{10}$

Shortest sides **Longest sides** **Remaining sides**

▶ Because the ratios are not equal, $\triangle ABC$ and $\triangle GHJ$ are not similar.

Since $\triangle ABC$ is similar to $\triangle DEF$ and $\triangle ABC$ is not similar to $\triangle GHJ$, $\triangle DEF$ is not similar to $\triangle GHJ$.

2. Which of the following three triangles are similar?



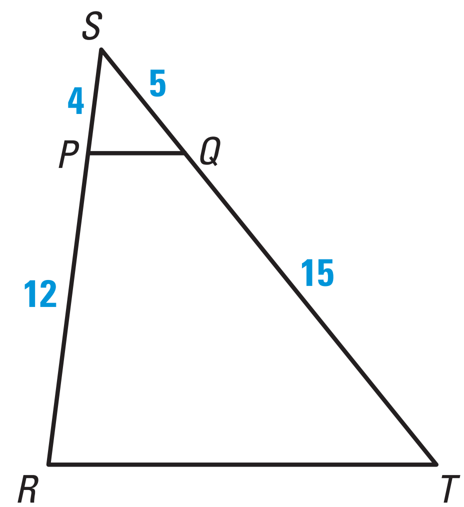
EXAMPLE 3 ***Using the SAS Similarity Theorem***

Use the given lengths to prove that $\triangle RST \sim \triangle PSQ$.

SOLUTION

GIVEN ▶ $SP = 4$, $PR = 12$, $SQ = 5$, $QT = 15$

PROVE ▶ $\triangle RST \sim \triangle PSQ$



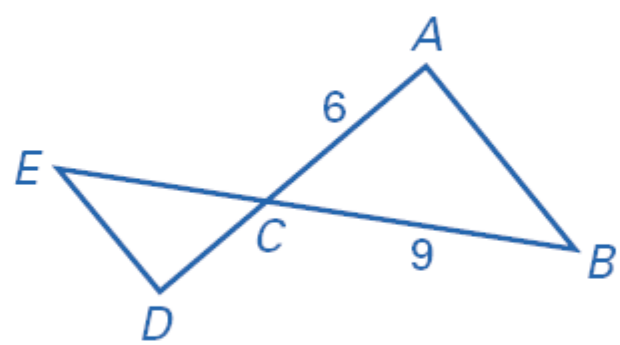
Paragraph Proof Use the SAS Similarity Theorem. Begin by finding the ratios of the lengths of the corresponding sides.

$$\frac{SR}{SP} = \frac{SP + PR}{SP} = \frac{4 + 12}{4} = \frac{16}{4} = 4$$

$$\frac{ST}{SQ} = \frac{SQ + QT}{SQ} = \frac{5 + 15}{5} = \frac{20}{5} = 4$$

So, the lengths of sides $\overline{SR}$ and $\overline{ST}$ are proportional to the lengths of the corresponding sides of $\triangle PSQ$. Because $\angle S$ is the included angle in both triangles, use the SAS Similarity Theorem to conclude that $\triangle RST \sim \triangle PSQ$.

3. If the figure AC = 6, AD = 10, BC = 9, & BE =15. Describe how to prove that $\triangle DCE$ is similar to $\triangle ACB$.



Draw the given triangles roughly to scale. If possible, name theorems that can be used to prove the triangles are similar.

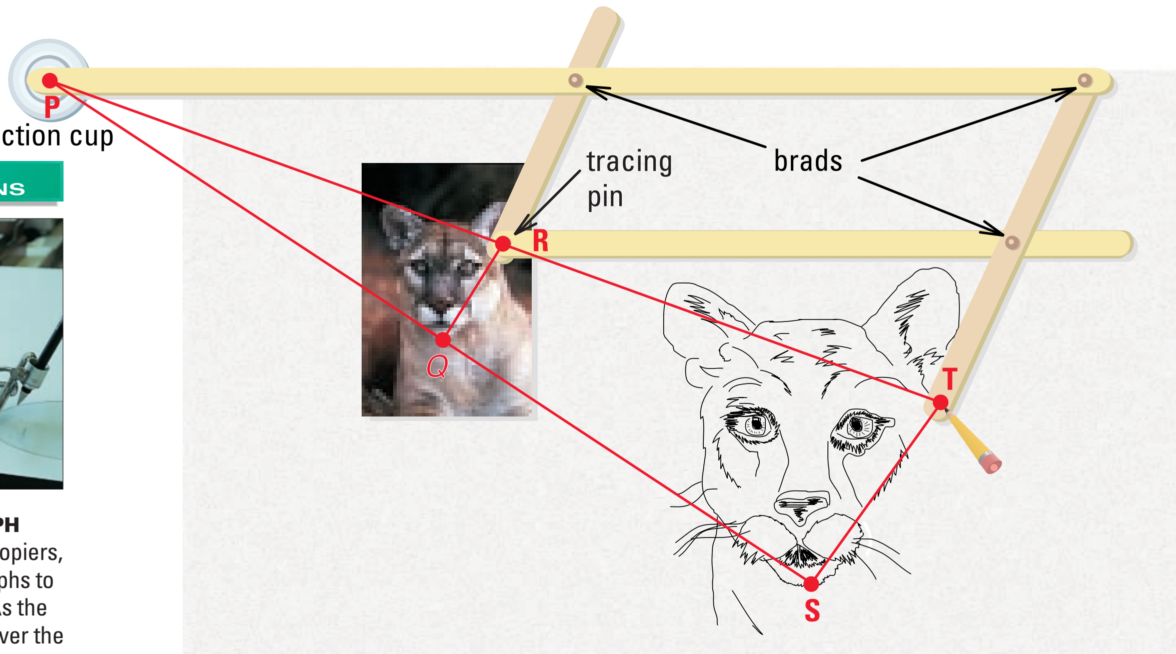
4. In $\triangle ABC$, AC = 5, BC = 20, & $m\measuredangle C = 90°$. In $\triangle DEF$, DF = 3, EF = 12, & $m\measuredangle F = 90°$.

5. In $\triangle XYZ$, XY = 5, YZ = 4, & $m\measuredangle Z = 50°$. In $\triangle UVW$, UV = 10, VW = 8, & $m\measuredangle W = 50°$.

6. In $\triangle PQR$, PQ = 22.5, QR = 12, & PR = 13.5. In $\triangle STU$, ST = 30, TU = 16, & SU = 18.

EXAMPLE 4 *Using a Pantograph*

SCALE DRAWING As you move the tracing pin of a *pantograph* along a figure, the pencil attached to the far end draws an enlargement. As the pantograph expands and contracts, the three brads and the tracing pin always form the vertices of a parallelogram. The ratio of PR to PT is always equal to the ratio of PQ to PS. Also, the suction cup, the tracing pin, and the pencil remain collinear.



FOCUS ON APPLICATIONS

PANTOGRAPH Before photocopiers, people used pantographs to make enlargements. As the tracing pin is guided over the figure, the pencil draws an enlargement.

a. How can you show that $\triangle PRQ \sim \triangle PTS$?

b. In the diagram, PR is 10 inches and RT is 10 inches. The length of the cat, RQ in the original print is 2.4 inches. Find the length TS in the enlargement.

SOLUTION

a. You know that $\frac{PR}{PT} = \frac{PQ}{PS}$. Because $\angle P \cong \angle P$, you can apply the SAS Similarity Theorem to conclude that $\triangle PRQ \sim \triangle PTS$.

b. Because the triangles are similar, you can set up a proportion to find the length of the cat in the enlarged drawing.

$\frac{PR}{PT} = \frac{RQ}{TS}$ **Write proportion.**

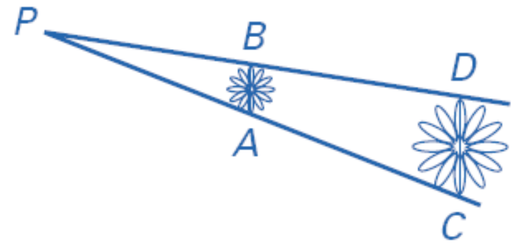
$\frac{10}{20} = \frac{2.4}{TS}$ **Substitute.**

$TS = 4.8$ **Solve for *TS*.**

▶ So, the length of the cat in the enlarged drawing is 4.8 inches.

A pantograph is used to draw an enlargement of a daisy. In the diagram, $\frac{PB}{PD} = \frac{PA}{PC}$.

7. Why is $\triangle PBA \sim \triangle PDC$?

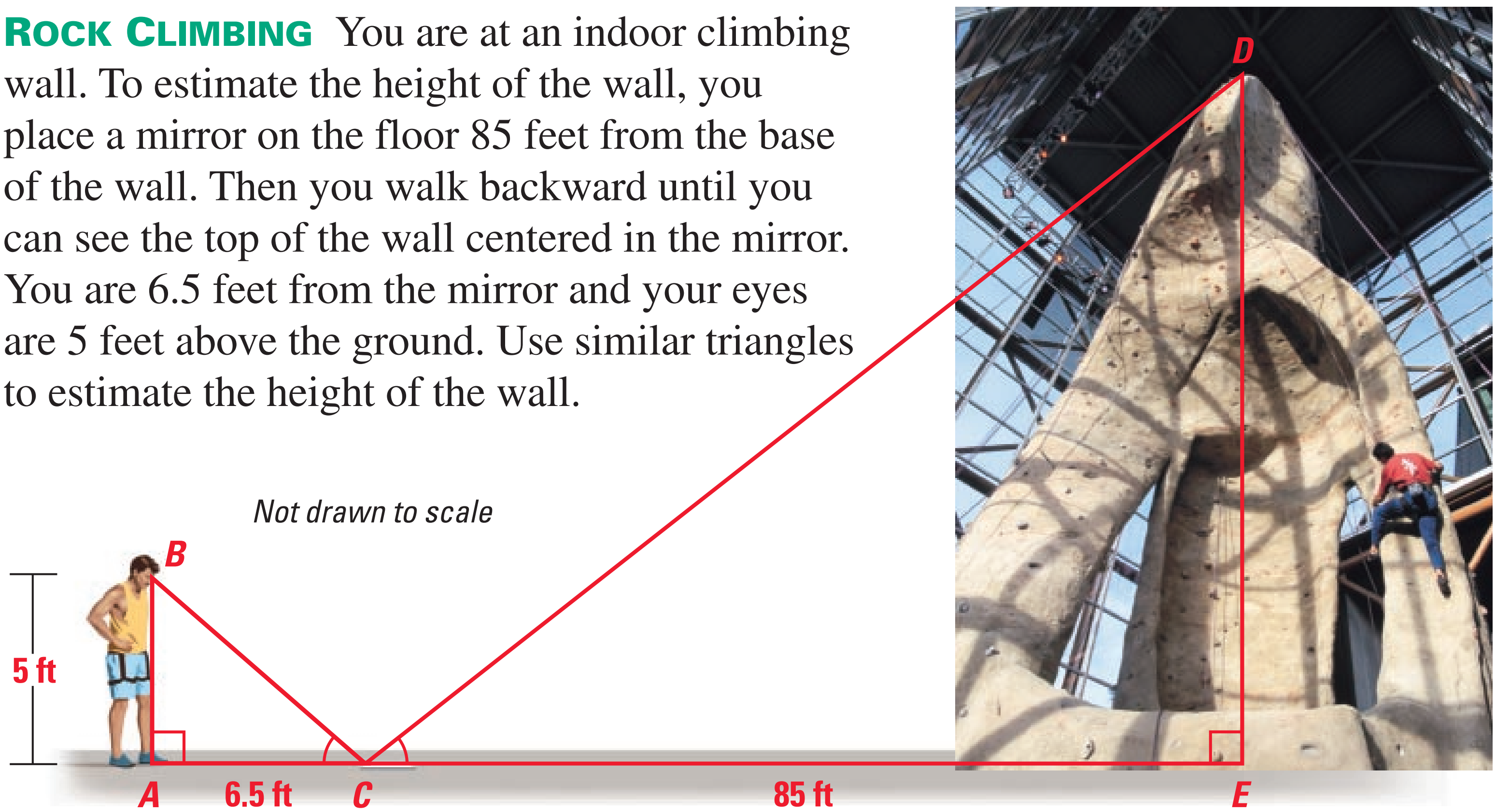


8. In the diagram, PA = 8in. and AC = 8 in. The diameter of the original daisy is 1.8 in. What is the diameter of the daisy in the enlargement?

9. Suppose you wanted the diameter of the enlarged daisy to be 4 in. What would the ratio $\frac{PA}{PC}$ have to be?

EXAMPLE 5 *Finding Distance Indirectly*

ROCK CLIMBING You are at an indoor climbing wall. To estimate the height of the wall, you place a mirror on the floor 85 feet from the base of the wall. Then you walk backward until you can see the top of the wall centered in the mirror. You are 6.5 feet from the mirror and your eyes are 5 feet above the ground. Use similar triangles to estimate the height of the wall.



SOLUTION

Due to the reflective property of mirrors, you can reason that $\angle ACB \cong \angle ECD$. Using the fact that $\triangle ABC$ and $\triangle EDC$ are right triangles, you can apply the AA Similarity Postulate to conclude that these two triangles are similar.

$\frac{DE}{BA} = \frac{EC}{AC}$ **Ratios of lengths of corresponding sides are equal.**

$\frac{DE}{5} = \frac{85}{6.5}$ **Substitute.**

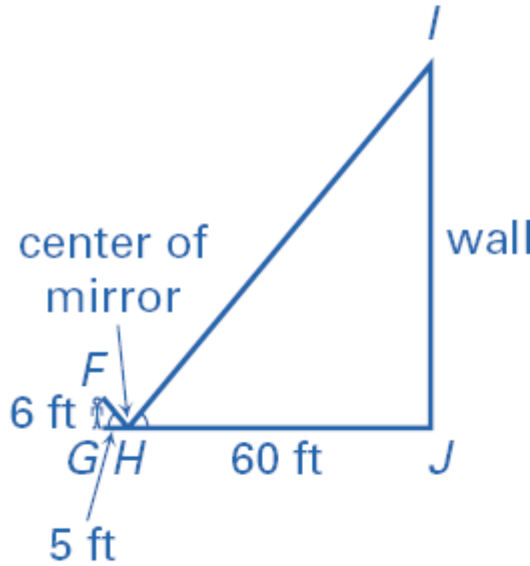
$65.38 \approx DE$ **Multiply each side by 5 and simplify.**

▶ So, the height of the wall is about 65 feet.

FOCUS ON APPLICATIONS

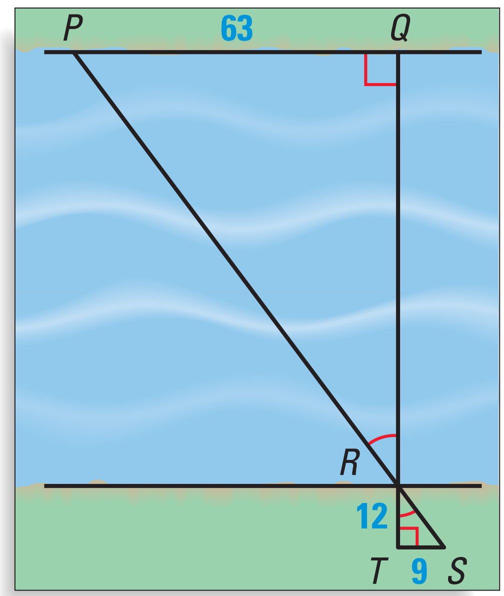
REAL LIFE **ROCK CLIMBING** Interest in rock climbing appears to be growing. From 1988 to 1998, over 700 indoor rock climbing gyms opened in the United States.

10. At an indoor climbing wall, a person whose eyes are 6 ft from the floor places a mirror on the floor 60 ft from the base of the wall. They then walk backwards 5 ft before seeing the top of the wall in the center of the mirror. Use similar triangles to estimate the height of this wall.



EXAMPLE 6 *Finding Distance Indirectly*

INDIRECT MEASUREMENT To measure the width of a river, you use a surveying technique, as shown in the diagram. Use the given lengths (measured in feet) to find RQ.



SOLUTION

By the AA Similarity Postulate, $\triangle PQR \sim \triangle STR$.

$$\frac{RQ}{RT} = \frac{PQ}{ST}$$ **Write proportion.**

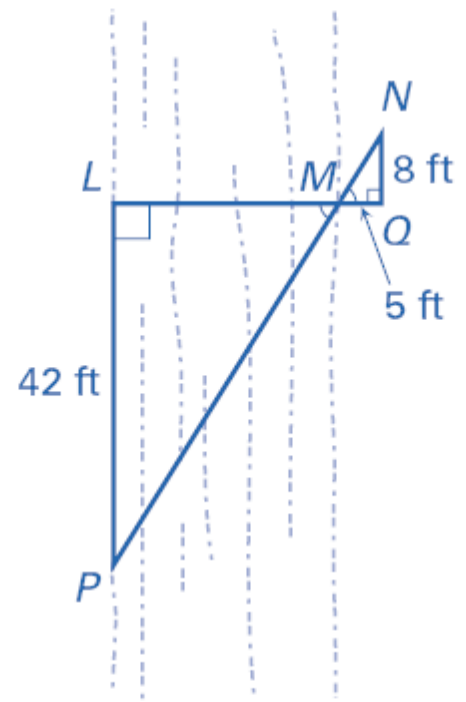
$$\frac{RQ}{12} = \frac{63}{9}$$ **Substitute.**

$$RQ = 12 \cdot 7$$ **Multiply each side by 12.**

$$RQ = 84$$ **Simplify.**

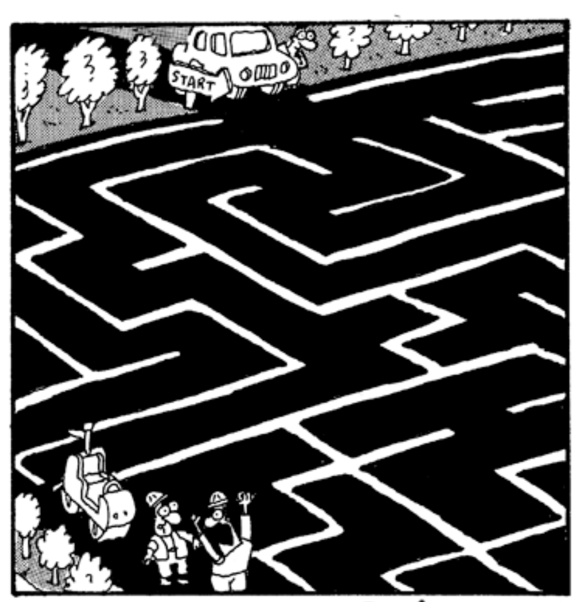
▶ So, the river is 84 feet wide.

11. Use the given lengths to find the width LM of the river below.



12. (Refer to problem 10) Suppose another person whose eyes are 5.5 ft from the floor places the mirror 50 ft from the base of this wall. If the wall is 65 ft high, how far from the mirror will the person need to stand to see the top of the wall centered in the mirror?

13. What are the two similarity theorems that we studied in this lesson?



Larry's career as a Parking Lot Painter comes to an abrupt end.